Catching Failures in Robust Phase Estimation (originally: Calibration for Single-Qubit Gates using Robust Phase Estimation)

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Quantum process characterization

If we want to do gate-based quantum computation...
$\Rightarrow$ Need to measure errors in a universal set.

Local, unitary errors (and also decoherence, crosstalk, etc.)

- Process tomography: inefficient for specific parameters, typically needs perfect SPAM
- Gate set tomography: powerful, few assumptions, but even more inefficient
- Randomized benchmarking: inefficient for specific parameters
- **Robust phase estimation**[1] (RPE): Heisenberg-limited, handles imperfect SPAM, nonadaptive, space-efficient

This work:

Supplementary methods to improve RPE performance.

[1] S. Kimmel, G. H. Low, and T. J. Yoder, Phys. Rev. A 92, 062315 (2015).

Robust phase estimation

Schematic for an RPE error estimation method:

- Apply composite operator k times to some initial state.
- Measure in one of a pair of bases (1 or 2).
- Design such that success probabilities are given by...

$$P_1(k) = \frac{1}{2} + \frac{1}{2}\cos(kA) + \delta_1(k),$$
$$P_2(k) = \frac{1}{2} + \frac{1}{2}\sin(kA) + \delta_2(k),$$

for A an invertible function of the desired error parameter, and $\delta_i(k)$ some additive deviations.

Robust phase estimation

Ideal success probabilities:

$$P_1(k) = \frac{1}{2} + \frac{1}{2}\cos(kA), \quad P_2(k) = \frac{1}{2} + \frac{1}{2}\sin(kA).$$

- Sample at $k_i = 2^i$ for $i = 0, 1, 2, \ldots$
- Take M_i measurements at k_i to obtain estimate $\hat{A}_i$ of A:

$$A \approx \hat{A}_i \text{ modulo } \frac{2\pi}{2^i},$$

with error $\sigma(\hat{A}_i) \propto \frac{1}{k_i\sqrt{M_i}} = \frac{2^{-i}}{\sqrt{M_i}}$.

Principal ranges:

Choose the (unique) $\hat{A}_i$ such that $|\hat{A}_i - \hat{A}_{i-1}| < 2^{-i}\pi$, for $i = 1, 2, 3, \ldots$

Catching failures in RPE

Drawback:

RPE error estimate assumes only statistical errors, but decoherent errors can systematically alter signal waveform.

Can cause effective parameter estimated to be different from desired parameter, in two ways:

- Effective parameter depends on sequence $\{k_i\}$: we show how to catch this!
- Effective parameter independent of sequence $\{k_i\}$: alteration to period of signal.

Catching failures in RPE

Idea:

Sample at second sequence of k values:

$$k_i' = k_i + f(i),$$

where $\frac{1}{2}f(i+1) \leq f(i) \leq 2^i$ for all i. Call resulting estimates $\hat{B}_i$.

Then for each i, check that $|\hat{B}_i - \hat{A}_i| \leq \frac{2\pi}{2^i}$:

- If false for some i, then

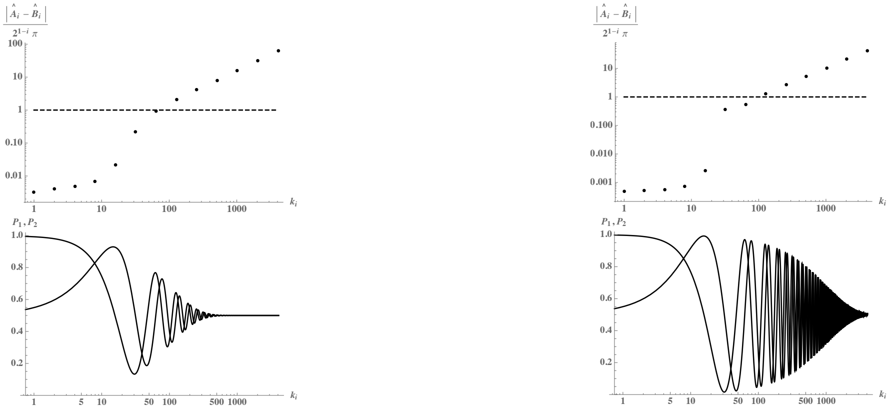
$$|\hat{A}_i - A| > \frac{\pi}{2^i} = \frac{\pi}{k_i}, \text{ or } |\hat{B}_i - A| > \frac{\pi}{2^i} > \frac{\pi}{k_i'} \quad \Rightarrow \quad \text{Failure!}$$

- If true for all i up to some J, then for effective target values A' and B',

$$P\left[|B' - A'| > \frac{4\pi}{2^J}\right] < \frac{2}{\sqrt{2\pi M_J}2^{M_J}}.$$

Catching failures in RPE

Example: Catching failures due to depolarization. Catch condition is $\frac{|\hat{B}_i - \hat{A}_i|}{2^{1-i}\pi} > 1$:



In these examples, $f(i) = 2^{i-1}$. Decay constants: $b = 0.1A$ (left), $b = 0.01A$ (right).

Advantages, Limitations, and Future Work

Advantages:

- Can avoid systematic errors that would otherwise throw off estimate.
- Does not require denser sampling than original protocol.

Limitations:

- Adversary who knows offset function f can fool us.
- Cannot catch alteration to period.

Future work:

- Optimize choice for f?
- Systematic error-specific methods to handle period alteration (some exist).

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